Kastberg Situations Draft
3/24/2009

Proportional Reasoning:

Prompt: Investigating proportional reasoning with middle school teachers resulted in a conflict over solutions to a work problem. The teachers felt two solutions to the following problem were mathematically reasonable.

> A crew of 8 people can build a concrete wall in 6 days. If four more people join the group from the beginning, how many days will it take to build the same wall?

One group of teachers shared that because 8 people can build a wall in 6 days, then 16 people can build a wall in 3 days. They used the following table.

People	2	4	8	12	16
Days	24	12	6		3

Reasoning that 12 is half way between 8 and 16, so the number of days should be half way between 6 and 3, the teachers identified 4.5 days as the answer.

Another group of teachers shared their table representing a relationship between the number of people and the number of days.

People	1	2	4	8	12
Days	48	24	12	6	

They reasoned that if you have four times the number of people there are ¼ as many days needed. If there are double the number of people there are ½ as many days needed. Using this approach since three times four people is twelve people, then 1/3 of 12 days, or 4 days must be the solution.

Commentary:

The mathematical issue centers on the development of counterexamples that are convincing using representations and units. The foci explore the representation of the relationship, use of units, and reasoning.

Mathematical Foci

Mathematical Focus 1:

Reasoning with units involves building an understanding of the original ratio in the context of the situation. Both the situation and the ratio imply a relationship.

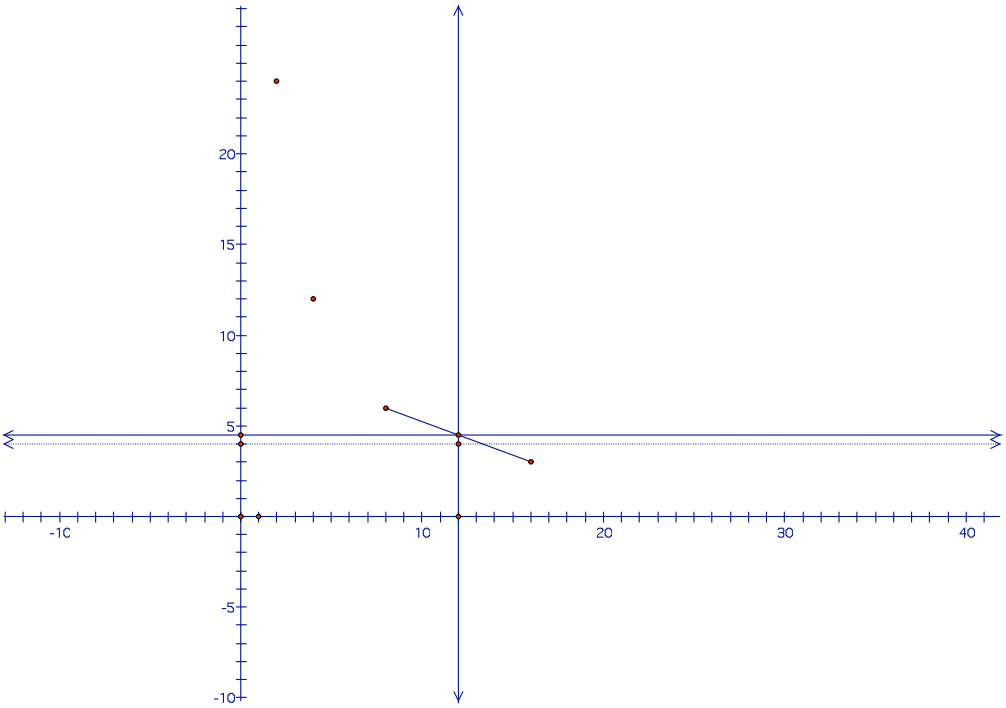
While the initial instinct is to use direct proportion because of the form of the problem, focus on relationships in the problem reveals the importance of units. In this problem 8 people work for six days to finish one job or 1/48 of the job is done by each person per day. If 12 people work

then ¼ of the job is done per day meaning the job takes 4 days. This reasoning builds from an understanding of 1 job as a constant. If the number of people is *x* and the number of days is *y*, then *x* people do 1/*y*, where *x* and *y* are not zero, of the job/day or *1*□□ of the job/day/person.

Mathematical Focus 2:

Representations as proportions.

The generation of a graph of the points may encourage the development of doubt regarding the use of linear reasoning in non-linear contexts. While the first group used non-linear reasoning in the development of some points in the table, their interpolated value relied on linear reasoning. The generation of a graph of both solutions and the data points generated by both groups provides another view that could disrupt the existing thinking of the groups. In the representation (12, 4) is on a line between two generated points and yet the balance of the data appears to curve.



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Linear Equations and Functions

Prompt: An algebra teacher had presented a method of generating linear functions in slope-intercept and standard form. Several students approached the desk with their functions in slope intercept form and asked "Do I just leave it like that or do I solve it?" She wondered "What does 'solve it' mean?"

Commentary

The mathematical issue centers on the difference between functional relationships with variables and equations with unknowns. The foci explore variation and functions. Equations in the context of linear functions are represented as special cases.

Mathematical Foci

Mathematical Focus 1:

Functional relationships can be manipulated to highlight different characteristics of a relationship.

Linear functions are used to represent a relationship between x and y such that there is a constant ratio when the ratio of the differences between y values and differences between x value for a pair of given points is created*:* $\square = \square 2 - \square 1 \square 2 - \square 1$ …..

Mathematical Focus 2:

Linear equations are special cases of linear functions for a given value or x or y.

When x or y is given it becomes possible to find the remaining unknown value using properties of the real numbers. For example in 5 = 12x – 2…..